Optimal Retirement Time and Consumption/Investment in Anticipation of a Better Investment Opportunity

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ABSTRACT

We study an optimal decision of an economic agent to retire from his job and live on invested wealth. A particular aspect of our problem is that, when the agent works as a wage earner his investment opportunity set is confined to a proper subset of all assets in the economy, but after retirement, his investment opportunity set becomes the full set of assets. The reason for this aspect is that before retirement he has only limited time and energy available for observing all market variables, and therefore, has full information about only a proper subset of all assets in the financial market. Hence he manages a portfolio consisting of only the assets in the subset according to the key behavioral assumption as in Merton (1987): an investor uses a security in constructing his optimal portfolio only if the investor knows about the security. But after retirement, he can contribute all his time and efforts of labor to perfecting his information about the financial market. For example, studying foreign financial markets or small companies may not be feasible during his time as a laborer, but after retirement he has full freedom and enough time to study these markets.

Mathematically, this problem can be modelled as a mixture of an optimal consumption and portfolio choice with two control variables and an optimal choice of a stopping time. We find an explicit solution and characterize optimal polices in an infinite-horizon continuous-time framework.

More concretely, our problem is as the following:

We consider an economic agent who has full information about only a proper subset of all assets in a market. This subset consists of a riskless asset and \( m \) risky assets. The set of all assets in the market consists of these and some other risky assets. The price processes of all risky assets in the market follows a geometric Brownian motion with constant market coefficients where the volatility matrix is invertible as in Merton’s problem. However they are not known to the agent until retirement. We assume that the risk-free rate is a constant \( r > 0 \) and the price \( p_0(t) \) of the riskless asset follows a deterministic process

\[
dp_{0}(t) = p_{0}(t)r dt, \quad p_{0}(0) = p_{0},
\]

as in Merton’s problem. The price \( p_{j}(t) \) of the \( j \)-th risky asset in the subset, as in Merton’s problem, follows a geometric Brownian motion

\[
dp_{j}(t) = p_{j}(t)\{\alpha_{j} dt + \sum_{k=1}^{m} \sigma_{jk} dw_{k}(t)\}, \quad p_{j}(0) = p_{j}, \quad j = 1, \ldots, m,
\]
where $w(t) = (w_1(t), \ldots, w_m(t))$ is an $m$-dimensional standard Brownian motion defined on the underlying probability space $(\Omega, \mathcal{F}, P)$. The market parameters, $\alpha_j$’s and $\sigma_{jk}$’s for $j, k = 1, \ldots, m$, are assumed to be constants. Let $(\mathcal{F}_t)_{t=0}^\infty$ be the augmentation under $P$ of the natural filtration generated by the standard Brownian motion $(w(t))_{t=0}^\infty$. We assume that the matrix $D = (\sigma_{ij})_{i,j=1}^m$ is nonsingular, i.e., there is no redundant asset among the $m$ risky assets as in Merton’s problem. Hence $\Sigma \equiv DD^T$ is positive definite. The problem in this paper is different from an ordinary consumption and investment problem in the following sense: the agent endowed with initial wealth $x$ can only manage the assets in the subset since he has full information only about assets in the subset and has no spare time to complete his information while he labors. However, he is able to manage the whole assets in the market after retiring from his job since he can devote his time and efforts to perfecting his information so that he knows the price processes of the whole risky assets in the market. Savage (1954) and Anscombe and Aumann(1963) provided axiomatic conditions on the agent’s preference under which we can assume the existence of such knowledge.

Let $R_1$ be the Sharpe ratio vector of the subset of assets and $R_2$ that of the whole assets in the market. As in Merton’s problem, we define nonnegative constants

$$\kappa_1 \equiv \frac{1}{2} \| R_1 \|^2 = \frac{1}{2} (\alpha - r1_m)^T \Sigma^{-1} (\alpha - r1_m)^T,$$

and

$$\tilde{\kappa}_2 \equiv \frac{1}{2} \| R_2 \|^2.$$

Of course the constant $\tilde{\kappa}_2$ is not known to the agent before retirement. However we assume that the agent has a partial prior information represented by a $\sigma$-algebra $\mathcal{I} \subset \mathcal{F}$ the probability distribution of $\tilde{\kappa}_2$ conditioned on which is known to him before retirement.

We obtain closed forms for the optimal retirement policy as well as for the optimal consumption and portfolio policy under a fairly general assumption that the agent has time-separable von Neumann-Morgenstern utility. We show that it is optimal to retire if and only if the agent’s wealth exceeds a critical level that is obtained from a free boundary value problem. A wage earner stops his work and becomes a full-time investor as soon as he becomes sufficiently wealthy, an intuitively appealing result. An interesting property is that the agent consumes less if the agent expects a better investment opportunity after retiring from labor than he does if he does not have such an option. Intuitively, he tries to accumulate his wealth fast enough to exploit a better investment opportunity and sacrifices his current consumption.

The optimal retirement problem can be investigated in the context of a real option. The economic agent has an option to invest in broader set of assets and the cost associated with exercising the option is foregone his wage income. He will exercise the option only when the benefit from exercising the option far exceeds the cost. As we said in the above, he will exercise the option only when he is sufficiently rich and the benefit of exploiting a better investment opportunity surpasses the cost of losing his wage income.

There has been extensive research in consumption and portfolio selection after Merton’s pioneering study (Merton 1969, 1971). Karatzas and Wang (2000) first studied a mixture of optimal stopping and optimal consumption and portfolio selection problems by a martingale method. Choi, Koo, and Kwak (2003) extended Karatzas and Wang’s results to the case where an economic agent has stochastic differential utility relying on the method of forward-backward stochastic differential equations. We use a dynamic programming method in this paper which is
different from the martingale method of the previous research on the mixture problem. We ex-

tend the solution method of a Hamilton-Jacobi-Bellman equation in Karatzas, Lehoczky, Sethi,
and Shreve (1986) to deal with the optimal retirement problem. In contrast with Karatzas and
Wang (2000) the horizon of optimization extends beyond the discretionary retirement time.

Recently there have been a few other studies on mixed consumption-portfolio-stopping prob-

lem. Jeanblac, Lakner and Kadam (2004) have solved a problem of an agent under obligation to
pay a debt at a fixed rate who can declare bankruptcy. Choi and Koo (2005) have studied the

effect of a preference change around a discretionary stopping time. Choi and Shim (2006) have
studied a problem in which a wage earner can choose consumption/investment policies, and
the time to retire considering a trade-off between income and disutility from labor by using
the dynamic programming method.

There is also a study which studies the effect of enlargement of the investment opportunity set

facing an economic agent. Choi, Koo, Shim, and Zariphopoulou (2003) have studied a con-

sumption and investment problem in which an agent’s investment opportunity set gets larger if
the agent’s wealth touches a critical level. The critical wealth level is given exogenously in their

model. However, it is endogenously determined as a result of an optimal retirement decision in

this paper.

In this paper there is no risk in wage income and we do not study effects of uninsurable in-

come risk. Uninsurable income risk has been investigated by Duffie, Fleming, Soner, and Za-

riphopoulou (1997), Koo (1998) and Cuoco (1999), etc. Here we focus on studying the optimal
consumption and portfolio selection of an agent who has the option to retire and become a

full-time investor with a better investment opportunity.

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